ABSORPTION POWER AND LINEWIDTHS IN QUANTUM WELLS WITH PÖSCHL-TELLER HYPERBOLIC POTENTIAL IN MAGNETIC FIELDS

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Abstract: Explicit expressions for magnetoconductivity and absorption power in hyperbolic quantum well with Pöschl-Teller potential type under the influence of a static magnetic fielf was obtained using the state-independent operator projection technique. The dependence of absorption power on photon energy was calculated and graphically plotted. From the graph of absorption power as a function of photon energy, we investigated the optically detected magneto-phonon resonance effect and the spectral linewidths of the resonance peaks. The obtained results show that the appearance of resonance peaks satisfies the law of conservation of energy, and the spectral linewidths of the resonant peaks vary with temperature, magnetic field intensity, and well parameters.
Keywords: absorption power, quantum well, hyperbolic, Pöschl-Teller potential, ODMPR, linewidths

1 INTRODUCTION

In recent years, studies on low-dimensional semiconductor physics have not stopped growing and obtained considerable achievements. Scientists have found many new effects in low-dimensional semiconductors with different types of confined potentials, such as electron - phonon resonance (EPR), magnetophonon resonance (MPR), cyclotron resonance (CR) [1, 2, 3, 4, 5]. However, most of these work considering the wells with the square or parabolic confined potentials. Recent technology in molecular-beam epitaxy growth techniques have enabled us to create potential profiles with various reasonable nonsquare shapes [6]. The properties of these nonsquare quantum wells (QWs) are specially focused due to their various applications in fabricating new optoelectronic devices.

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In this paper we consider an nonsquare QW structure whose potential profile is described by a modified hyperbolic shape quantum well with Posch - Teller type. We consider the MPR effect, clarify the nature of the optically detected MPR (ODMPR) effect. The dependences of the spectral linewidths of the ODMPR peaks on temperature, magnetic field, and well parameter are also investigated using the Profile method by means of Mathematica software [7, 8].

The paper is organized as follows: model and theoretical framework is described in Section 2, results and discussions are presented in Section 3, and conclusions are given in Section 4.

2 MODEL AND THEORY

We consider a Pöschl-Teller hyperbolic quantum well with the confined potential expressed in the form [9]

$$V(z) = \frac{V_1 - V_2\cosh(\alpha z)}{\sinh^2(\alpha z)}, \tag{1}$$

where V_1, V_2 and α are well parameters.

The energy spectrum of electron in the well, subjected to a magnetic field along z axis with Landau gauge $\vec{A} \equiv (0, xB, 0)$, gets the form

$$E_{N,n} = \left(N + \frac{1}{2}\right)\hbar\omega_c + E_n, \tag{2}$$

with E_n is the energy corresponding to the z-axis, having the form [9]

$$\begin{aligned}
E_n &= -\frac{\alpha^2}{8}[\upsilon - \mu - (1+2n)]^2, \\
n &= 0, 1, 2, ..., \left[\frac{1}{2}(\upsilon - \mu - 1)\right], \\
\mu &= \frac{1}{2\alpha}\sqrt{8(V_1+V_2)+\alpha^2};\ \upsilon = \frac{1}{2\alpha}\sqrt{8(V_1-V_2)+\alpha^2},
\end{aligned}$$

$\omega_c = eB/m^*$ is the cyclotron frequency. The corresponding wave function is [9]

$$\Psi_{N,n,k_y} = (2^N N!\sqrt{n} l_m)^{-1/2}\exp(ik_y y)\psi_N(x - x_0)\varphi_n(z), \tag{3}$$

where N = 0,1,2, is the Landau level index; $\ell_m = (\hbar/eB)^{1/2}$ is the magnetic length; $\psi_N(x - x_0)$ is the harmonic oscillator wave function centered at $x_0 = \frac{1}{eB}(-i\hbar)\frac{\partial}{\partial y}$, k_y is electron wave vector in y-direction; $\varphi_n(z)$ is the electron wave function in z-direction:

$$\varphi_n(u) = Cu^{\delta}(1-u)^{\varepsilon}{}_2F_1\left[-n, n + 2(\delta + \varepsilon + 1/4); 2\delta + 1/2; u\right], \tag{4}$$

with u is given by $u = \tanh^2(\alpha z/2\ )$. The normalized constant C can be expressed in term of Gamma function:

$$C = \sqrt{2\alpha\varepsilon\Gamma\left(n+\mu+1\right)\Gamma\left(n+\mu+2\varepsilon+1\right)/n!\Gamma(\mu+1)^2\Gamma(n+2\varepsilon+1)}.$$

Using the operator projection method, we find the expression for absorption power as follows:

$$P\left(\omega\right) = \frac{E_0^2}{2\hbar\omega}\sum_{\alpha}\frac{\left|j_\alpha^+\right|^2(f_\alpha - f_{\alpha+1})B(\omega)}{\left(\omega-\omega_c\right)^2 + \left(B(\omega)\right)^2}, \tag{5}$$

where $|j_\alpha^+| = \left|\langle\alpha+1|\, j^+\, |\alpha\rangle\right|^2 = (N+1)(2e^2\hbar\omega_c)/m^*$; f_α and $f_{\alpha+1}$ are the Fermi-Dirac distribution function of electron at state $|\alpha\rangle = |N, n, k_y\rangle$ and $|\alpha+1\rangle = |N+1, n, k_y\rangle$, respectively. The relaxation rate $B(\omega)$ is given by

$$\begin{aligned} B\left(\omega\right) = {} & \sum_{N',n'}\frac{e^2\hbar\omega_{LO}}{2\Omega\varepsilon_0}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\frac{1}{f\left(N,n\right) - f\left(N+1,n\right)} \\ & \times\int\limits_{-\infty}^{\infty} G_{n,n'}\left(q_z\right)dq_z\int\limits_0^{\infty} dq_\perp\frac{q_\perp^3}{\left(q_\perp^2+q_d^2\right)^2}K(N,N';t) \\ & \times\left[\left(\left(1+N_q\right)f\left(N,n\right)\left\{1-f\left(N',n'\right)\right\} - N_q f\left(N',n'\right)\left\{1-f\left(N,n\right)\right\}\right)\times\delta(M_1^-)\right. \\ & + \left(N_q f\left(N,n\right)\left\{1-f\left(N',n'\right)\right\} - (1+N_q)f\left(N',n'\right)\left\{1-f\left(N,n\right)\right\}\right)\times\delta(M_1^+) \\ & + \left(\left(1+N_q\right)f\left(N',n'\right)\left\{1-f\left(N+1,n\right)\right\} - N_q f\left(N+1,n\right)\left\{1-f\left(N',n'\right)\right\}\right)\times\delta(M_2^-) \\ & + \left(N_q f\left(N',n'\right)\left\{1-f\left(N+1,n\right)\right\} - (1+N_q)f\left(N+1,n\right)\left\{1-f\left(N',n'\right)\right\}\right)\times\delta(M_2^+), \end{aligned} \tag{6}$$

where

$$M_1^{\pm}\left(N,N',n,n'\right) = \hbar\omega + \left(N-N'\right)\hbar\omega_c \pm \left|\left(E_{n_z}^{(2)} - E_{n_z}^{(1)}\right)\right| \pm \hbar\omega_q, \tag{7}$$

$$K\left(N_\alpha,N_\eta,t\right) = \frac{N_{\min}!}{N_{\max}!}t^{N_\eta-N_\alpha}e^{-t}\left[L_{N_\alpha}^{N_\eta-N_\alpha}\left(t\right)\right]^2,$$

with $L_{N_\alpha}^{N_\eta-N_\alpha}$ is Laguerre polynomical, $t \equiv \hbar\left(q_x^2+q_y^2\right)/2eB$, $N_{\min} = \min\{N_\eta\ , N_\alpha\}$ and $N_{\max} = \max\{N_\eta, N_\alpha\}$, $G_{n,n'}$ is the form factor,

$$M_2^{\pm}\left(N,N',n,n'\right) = \hbar\omega + \left(N'-N-1\right)\hbar\omega_c \pm \left|\left(E_{n_z}^{(2)} - E_{n_z}^{(1)}\right)\right| \pm \hbar\omega_q. \tag{8}$$

The Dirac delta functions ($\delta(M_\ell^{\pm}), \ell = 1, 2$) in Eq. (6) are replaced by Lorentzian functions of widths $\eta_{N,N'}^+$ and $\eta_{N+1,N'}^+$, namely

$$\delta\left(M_{1,2}^{\pm}\right) = \frac{1}{\pi}.\frac{\hbar\eta_{N,N'}^{\pm}}{\left(M_{1,2}^{\pm}\right)^2 + \hbar^2\left(\eta_{N,N'}^{\pm}\right)^2}, \tag{9}$$

where

$$\left(\eta^{\pm}_{NN'}\right)^2 = \frac{e^2\omega_{LO}}{16\hbar\pi^2\varepsilon_0}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}\right)\left(N_q+\frac{1}{2}\pm\frac{1}{2}\right)\int\limits_{-\infty}^{\infty} G_{n,n'}(q_z)dq_z\int\limits_{0}^{\infty} dq_\perp\frac{q_\perp^3}{\left(q_\perp^2+q_d^2\right)^2}K(N,N';t).$$

We can see that these analytical results appear very involved. However, physical conclusion can be drawn from numerical computations and graphical representation.

3 NUMERICAL RESULTS AND DISCUSSIONS

The condition for optically detected magnetophonon resonance (ODMPR) can be expressed in the form

$$\hbar\omega = \pm(N'-N)\hbar\omega_c \pm (E_{n'}-E_n) \pm \hbar\omega_{LO}.$$

When this condition is satisfied, electrons transit between two Landau levels with indexes N, N' and two subband energy levels E_n, E_n' by absorbing or emitting a photon of energy $\hbar\omega$, accompanied with the absorption or emission of longitudinal optical phonons with energy $\hbar\omega_{LO}$. In the case of absence the intersubband transition, the condition becomes:

$$\hbar\omega = \pm(N'-N)\hbar\omega_c \pm \hbar\omega_{LO}.$$

The obtained analytical results can be clarified by numerical computation and graphical plotting for a specific GaAs/AlAs with parameters used are [10, 11]: high frequency dielectric constants $\chi_\infty = 10.9$, static frequency dielectric constants $\chi_0 = 12.9$, vacuum dielectric constants $\varepsilon_0 = 12.5$, $E_0 = 10^5$ V/m, effective mass of electron $m^* = 6.097\times10^{-32}$, Planck constants $\hbar = 6.625\times10^{-34}/(2\pi)$ Js, Boltzmann constants $k_B = 1.38066\times10^{-23}$ J/K, frequency of longitudinal optical phonon $\omega_{LO} = 36.25\times1.6\times10^{-22}/\hbar$, $n=1$, $n'=n+1$.

Figure 1 shows the graph describing the absorption power $P(\omega)$ as a function of photon energy. From the graph we can see four peaks, satisfying different resonance conditions:

+ Peak 1 appears at the location of the photon energy $\hbar\omega = 34.574$ meV. This value is exactly equal to the cyclotron energy $\hbar\omega_c$, therefore it represents the cyclotron resonance.

+ Peaks 2 and 4 appear in turn at two positions $\hbar\omega = 70.642$ meV and $\hbar\omega = 143.103$ meV, satisfying the condition $\hbar\omega = \hbar\omega_c + E_\beta - E_\alpha \mp \hbar\omega_{LO}$ or $\hbar\omega = 34.574 + 72.279 \mp 36.25$ meV. These peaks describe ODMPR effect with electron intersubband transitions.

+ Peak 3 can be found at the photon energy $\hbar\omega$ =106.892 meV, satisfying condition

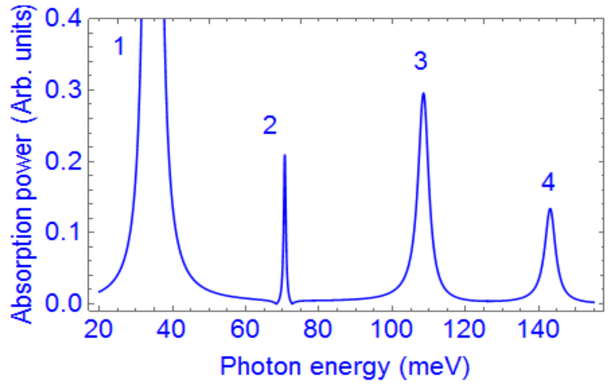

Figure 1: *Absorption power* $P(\omega)$ *as function of the photon energy at* $T = 300\ K$, $B = 20\ T$, $\alpha = 2.52 \times 10^8 m^{-1}$.

$\hbar\omega = \hbar\omega_c + (E_\beta - E_\alpha)$. This is the condition for the cyclotron resonance with electron intersubband transition.

Figure 2(a) shows the dependence of the absorption power on the photon energy at different values of temperature. From the figure we can see that ODMPR peaks locate at the same position ($\hbar\omega$ =143.18 meV) and the linewidths increase with the temperature as shown in Fig. 2(b). This can be explained that as temperature increases, the probability of electron-phonon scattering increases, and so do the linewidths.

Figure 3(a) describes the absorption power as a function of the photon energy at three values of the magnetic field. The graph shows that when the magnetic field B increases, the positions of the resonance peak move toward the greater photon energy. This can be explained that when B increases, the cyclotron energy $\hbar\omega_c$ increases, the photon energy corresponds to the ODMPR condition $\hbar\omega = \hbar\omega_c + (E_\beta - E_\alpha) + \hbar\omega_{LO}$ increases, so that the resonance peak will shift toward the higher energy values.

Graph 3(b) shows the dependence of ODMPR peak linewidths on the magnetic field. From the figure we can see that the linewidths increase as the magnetic field rises. This can be explained that when the magnetic field rises, the cyclotron radius $l_B = \sqrt{\frac{\hbar}{eB}}$ decreases, causing the increase of electron confinement. This leads to the increase of probability of electron-phonon scattering. Therefore, the spectral linewidths increase with the magnetic field.

Graph 4(a) shows the dependence of the absorption power on the photon energy with different values of well parameter α at T = 300 K. From the graph we can see that when α parameter increases, the positions of the resonance peaks move toward the greater energy. This can be explained that when α increases, the energy difference $E_\beta - E_\alpha$ increases, therefore the photon energy corresponding to the

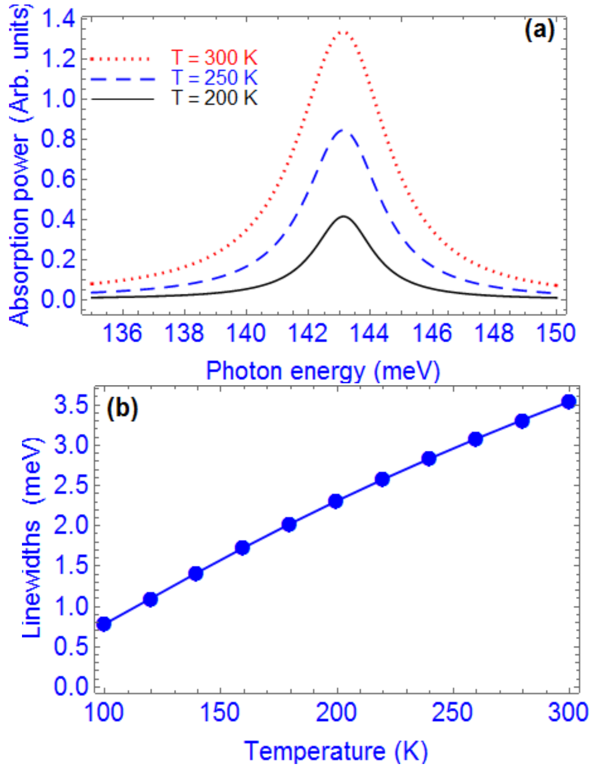

Figure 2: (a) *Dependence of absorption power* $P(\omega)$ *on photon energy at different values of temperature.* (b) *Dependence of the ODMPR peak linewidths on the temperature.*

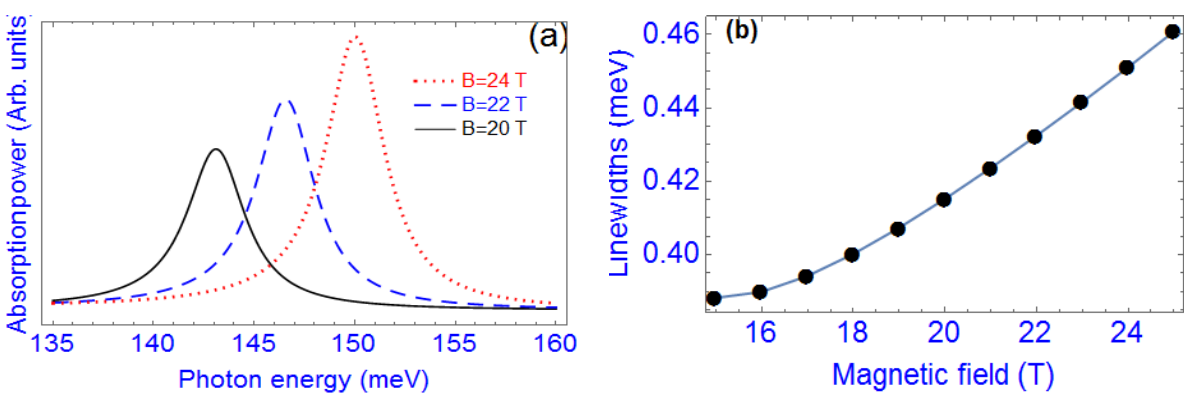

Figure 3: (a) *Dependence of absorption power* $P(\omega)$ *on photon energy at different values of magnetic field strength* B. (b) *Dependence of ODMPR peak linewidths on the magnetic field strength* B.

ODMPR condition $\hbar\omega = \hbar\omega_c + E_\beta - E_\alpha \pm \hbar\omega_{LO}$ condition increases. This is the reason why the resonance peaks move toward the higher energies.

Graph 4(b) shows the dependence of linewidths on the parameter α. We can see from the graph that the linewidths decreases as α increases. This can be explained

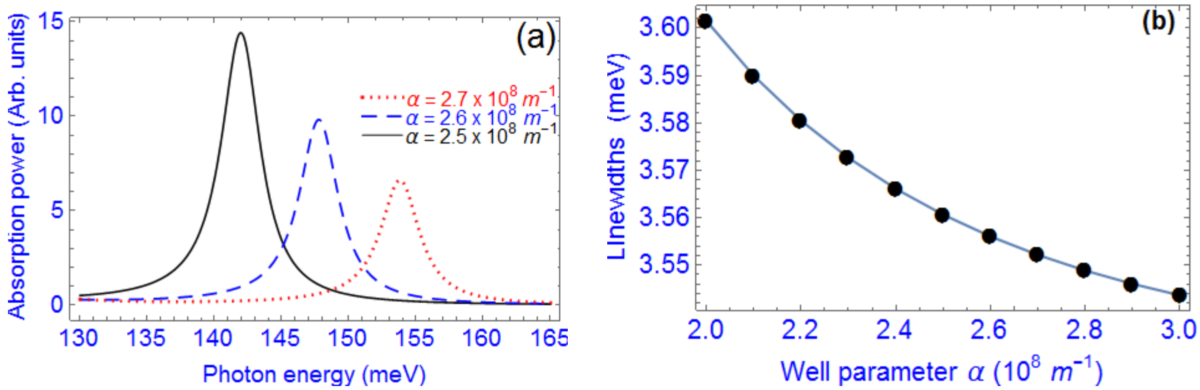

Figure 4: (a) *Dependence of absorption power* $P(\omega)$ *on photon energy at different values of well parameter* α. (b) *Dependence of the ODMPR peak linewidths on well parameter* α.

that when the α parameter increases, the confined potential decreases, resulting in the decrease of electron - phonon scattering probability. Therefore, the linewidths of the ODMPR peak decrease.

4 CONCLUSIONS

So far, we have derived the absorption power $P(\omega)$ for 2D electron gas in a quantum well with Pöschl-Teller hyperbolic potential, subjected to a periodic electric field and a static magnetic field. We obtained the MPR conditions and an energy range in which the relaxation rates are allowed. From the graph describing the absorption power $P(\omega)$ as a function of photon energy, we found four peaks, satisfying the general resonance condition $\hbar\omega = \pm(N' - N)\hbar\omega_c \pm (E_{n'} - E_n) \pm \hbar\omega_{LO}$. Considering the behaviour of ODMPR peaks we found that linewidths of ODMPR peak increase with the temperature and magnetic field, but decrease with the well parameter. The obtained results are explained satisfactorily.

In this work, we have not taken into account the influence of the spin - orbit coupling and the Zeeman spin splitting. Despite the above shortcomings of the theory, we expect that our results will help to understand the ODMPR effects in quantum well structures. Unfortunately, these results cannot be verified experimentally yet because no experimental results are available at the moment. However, we hope that these our results are helpful in future experimentation.

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