INFLUENCE OF PHONON CONFINEMENT ON CYCLOTRON–PHONON RESONANCE IN INFINITE SQUARE QUANTUM WELL

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Abstract: Magneto-optical absorption coefficient (MOAC) in infinite square quantum well (SQW) is investigated by time-dependent perturbation method. The dependence of MOAC on the photon energy is calculated and graphically plotted. From the graphs showing this dependence, we obtained the spectral line width (the full-width at half maximum - FWHM) of the cyclotron-phonon resonance peaks using Profile method. The results show that the occurrence of peaks satisfies cyclotron-phonon resonance conditions and the FWHM of resonance peak increases with the magnetic field and decreases with the width of the quantum well.
Keywords: Quantum well, Infinite square potential, Magneto-optical absorption coefficient, cyclotron - phonon resonance, Full-width at half maximum.

1. INTRODUCTION

Cyclotron-phonon resonance (CPR) is an effect resulting from the interaction between electrons and phonons in semiconductors subjected to an electromagnetic field and a static magnetic field [1, 2]. This effect is the combination of two types of resonances: the cyclotron resonance, where the electromagnetic wave frequency Ω is equal to cyclotron frequency ω_c [3, 4, 5] and the magneto-phonon resonance, where the phonon frequency ω_q is equal to ω_c [6, 7]. CPR is considered to be more general than the two above - mentioned resonances due to the combination of two actions: the absorption of a photon of energy $\hbar\Omega$ occurs simultaneously with absorbing or emitting a phonon. This means that the CPR arises at the photon energy $\hbar\Omega = p\hbar\omega_c \pm \hbar\omega_0$, where p is a positive integer. Because of its pivotal role, the CPR has attracted the attention of many physicists in theory [8, 9, 10] and experimentation [11, 12]. In theory, it is possible to study the CPR by deriving the magneto-optical absorption coefficient or power of electromagnetic wave.

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CPR effect has been studied both theoretically and experimentally in the case of bulk phonons (3D phonons). The influence of phonon confinement on electron-phonon interaction in general and CPR effect in particular is also investigated by introducing three phonon confinement models namely the slab model [13, 14, 15], guide mode [16] and Huang - Zhu [17].

In the present paper, we study the CPR effect due to electron - longitudinal optical phonon (LO phonon) confined in an infinite SQW subjected to an external electromagnetic field and a static magnetic field. The confinement of phonon obey the Huang - Zhu model. The dependence of FWHM on magnetic field strength, well width is examined by Profile method using Mathematica software.

2. EXPERSSION OF ABSORPTION COEFFICIENT OF ELECTROMAGNETIC WAVE IN INFINITE SQUARE QUANTUM WELL

We consider a symmetric infinite SQW in which electrons moves freely in xOy plane and are confined in z direction. The external magnetic field is applied along z direction ($\vec{B} = (0, 0, B)$). Landau gauge is chosen as $\vec{A} = (0, Bx, 0)$. The wave function and energy of electron take the forms

$$\psi\left(\vec{r}_\perp, z\right) = \psi_{N,n,k_y}\left(x, y, z\right) = \frac{1}{\sqrt{L_y}}\phi_N\left(\frac{x - \ell_B^2 k_y}{\ell_B}\right)e^{ik_y y}\psi_n\left(z\right), \tag{1}$$

$$E_{N,n} = E_N + E_n = \left(N + \frac{1}{2}\right)\hbar\omega_c + n^2 E_0, \tag{2}$$

where $\omega_c = eB/m$ is cyclotron frequency, $\ell_B^2 = \hbar/(m\omega_c)$, $E_0 = \hbar^2\pi^2/2m^* L_z^2$. The wave functions $\psi_n(z)$ and $\phi_N(x)$ are given by

$$\psi_n\left(z\right) = \sqrt{\frac{2}{L_z}}\sin\left(\frac{n\pi z}{L_z} + \frac{n\pi}{2}\right), \tag{3}$$

$$\phi_N\left(x\right) = \left(2^N N! \sqrt{\pi}\ell_B\right)^{-1/2}\exp\left[-\frac{\left(x - x_0\right)^2}{2\ell_B^2}\right]H_N\left[\frac{x - x_0}{\ell_B}\right], \tag{4}$$

where $x_0 = -\ell_B^2 k_y$.

The MOAC due to electron - phonon interaction is [18, 19]

$$K(\Omega) = \frac{\sqrt{\varepsilon}}{nc}\sum_i W_i f_i, \tag{5}$$

where ε and n are the dielectric constant and index of refraction, respectively, of the sample; f_i is the electron distribution function and W_i is the transition probability. The sum is taken over all the initial states i of electrons. The absorption coefficient is related to the transition probability for an electron to make transition by absorbing a photon and simultaneously absorbing or emitting a phonon. This transition probability is given by the second-order golden rule approximation [20]

$$W_i = \frac{2\pi}{\hbar} \sum_f | \langle f|M|i\rangle |^2 \times \delta(E_f - E_i \mp \hbar\omega_q \mp \hbar\Omega), \tag{6}$$

where the upper (-) and lower sign (+) refer to the phonon absorption and phonon emission, respectively, $\langle f|M|i\rangle$ is the transition matrix element for the interaction

$$\langle f|M|i\rangle = \sum_\alpha \frac{\langle f|H_{rad}|\alpha\rangle\langle\alpha|H_{int}|i\rangle}{E_i - E_\alpha - \hbar\omega_q} + \frac{\langle\alpha|H_{rad}|i\rangle\langle f|H_{int}|\alpha\rangle}{E_i - E_\alpha - \hbar\Omega}, \tag{7}$$

where H_{rad} is the Hamiltonian for the interaction between the electrons and radiation field, and H_{int} is the scattering potential due to the electron-phonon interaction. The sum is over all intermediate states $|\alpha\rangle$ of electron; E_i and E_f are the initial and final energies of electron; the photon and phonon energies are $\hbar\Omega$ and $\hbar\omega_q$, respectively.

The Hamiltonian for the interaction between the radiation field and electrons is

$$H_{rad} = -\frac{e}{m^*}\left(\frac{2\pi n\hbar}{\Omega\epsilon V}\right)^{1/2} \vec{\epsilon}\cdot\vec{P}, \tag{8}$$

where $\vec{\epsilon}$ is the polarization vector of the radiation field and $\vec{P}$ is the kinetic momentum operator.

Using the wave function in Eq. (1) and assuming that the electromagnetic field is linearly polarized transverse to the magnetic field, the matrix elements for photon absorption can be written as

$$|M_{rad}|^2 = |\langle N+1|H_{rad}|N\rangle|^2 = \left(\frac{e\hbar}{m^*}\right)^2 \left(\frac{2\pi n}{\hbar\Omega\epsilon V}\right)\left(\frac{\hbar m^*\omega_c}{2}\right)(N+1). \tag{9}$$

The matrix elements for interaction between electron and confined LO phonon can be expressed as follows [20]:

$$\begin{aligned}|M_{int}|^2 &= \left|\left\langle k_y', N', n'\left|H_{int}\right| k_y, N, n\right\rangle\right|^2 \\ &= \sum_{\vec{q}} \left|V_m\left(\vec{q}_\perp\right)\right|^2 \left|J_{NN'}\left(u\right)\right|^2 \left|G_{nn'}^{m\alpha}\right|^2 \left|t_{m\alpha}\left(q_\perp\right)\right|^2 \left(N_{LO} + \frac{1}{2} \pm \frac{1}{2}\right)\delta_{k_y+q_y,k_y'},\end{aligned}$$

where

$$\left|V_m\left(q_\perp\right)\right|^2 = \frac{e^2\hbar\omega_{LO}}{\varepsilon L_z}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right)\left(q_\perp^2 + \frac{m^2\pi^2}{L_z^2}\right)^{-1} \quad m = 1, 2, 3...,$$

$$|J_{NN'}(u)|^2 = \frac{n_2!}{n_1!} u^{n_1-n_2} e^{-u} [L_{n_2}^{n_1-n_2}]^2, \tag{10}$$

with $u = \ell_B^2 q_\perp^2/2$, $n_1 = \max(N, N')$, $n_2 = \min(N, N')$, $L_{n_2}^{n_1-n_2}$ is the associated Laguerre polynomial, $G_{nn'}^{m\alpha}$ is given by

$$G_{nn'}^{m\alpha} = \int_{-L_z/2}^{L_z/2} \psi_n'(z)\, u_{m\alpha}(z)\, \psi_n(z)\, dz. \tag{11}$$

In (11) $u_{m\alpha}$ is the displacement of confined phonon oscillation along z direction. For the HZ model $u_{m\alpha}$ is given $u_{m+}(z) = \sin\left[\frac{\mu_m \pi z}{L_z}\right] + \frac{C_m z}{L_z}$ for odd modes $(m = 3, 5, 7, ...)$ and $u_{m-}(z) = \cos\left[\frac{m\pi z}{L_z}\right] - (-1)^{m/2}$ for even modes $(m = 2, 4, 6, ...)$; μ_m is the solution of $\tan(\mu_m \pi/2) = \mu_m \pi/2, m-1 < \mu_m < m;$ $C_m = -2\sin(\mu_m \pi/2)$. The term $t_{m\alpha}(q_\perp)$ is given by

$$\begin{aligned}
t_{m-}(q_\perp) &= \left[3q_\perp^2 + \frac{n^2\pi^2}{L_z^2}\right]^{-1/2}, \quad m = 2, 4, 6, ...,\\
t_{m+}(q_\perp) &= \left\{\left[1 + C_m^2\left[\frac{1}{6} - \mu_m^{-2}\pi^{-2}\right]\right] q_\perp^2 + (\mu_m^2\pi^2 - C_m^2) L_z^{-2}\right\}^{-1/2}, \quad m = 3, 5, 7....
\end{aligned}$$

The overlap integral in (11) can easily be evaluated for intrasubband and intersubband transitions. In the HZ model only even modes contribute for intrasubband transitions and odd modes contribute for intersubband transitions [13]. The transition probability is now becomes

$$\begin{aligned}
W_i = {} & \frac{2\pi}{\hbar}\left(\frac{V_0}{2\pi L_z}\right)^2 \frac{e^2\hbar\omega_{LO}}{\varepsilon L_z}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right)\frac{1}{\ell_B^2}\left|G_{nn'}^{m+}\right|^2 \\
& \times \sum_{N'}\sum_m \int_0^{+\infty} dq_\perp q_\perp \left(q_\perp^2 + \frac{m^2\pi^2}{L_z^2}\right)^{-1} \left|J_{NN'}(q_x)\right|^2 \left|t_{m\alpha}(q_\perp)\right|^2 \\
& \times \left(N_{LO} + \frac{1}{2} \pm \frac{1}{2}\right)\delta_{k_y+q_y,k_{y'}}\,\delta\left(E_f - E_i \mp \hbar\omega_{LO} \mp \hbar\Omega\right),
\end{aligned} \tag{12}$$

The Fermi - Dirac function in the presence of magnetic field takes the following form in the case of non-degenerate electron gas [13]

$$f_{Nn} = \frac{2n_e\pi\ell_B^2 L_z}{\xi} e^{-E_{N,n}/k_BT}, \tag{13}$$

where $E_{N,n} = (N + 1/2)\hbar\omega_c + E_n$ and $\xi = \sum\limits_{N,n} e^{-E_{N,n}/k_BT}$.

Doing some calculations and insert Eq. (12) and Eq. (13) into Eq. (5), we obtain the analytical expression of MOAC in the case of confined LO phonon

$$K(\Omega) = \frac{1}{\xi}\frac{e^2 n_e V_0^3 \omega_{LO}}{2\pi n_0 c\sqrt{\varepsilon} L_z^3 \ell_B^2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right)\sum_N \sum_{N'} \sum_m e^{-E_{N,n}/k_B T}$$
$$\times \left|G_{nn'}^{m\alpha}\right|^2 \int_0^{+\infty} dq_\perp q_\perp \left(q_\perp^2 + \frac{m^2\pi^2}{L_z^2}\right)^{-1} \left|J_{NN'}(u)\right|^2 \left|t_{m\alpha}(q_\perp)\right|^2$$
$$\times \left(N_{LO} + \frac{1}{2} \pm \frac{1}{2}\right)\delta_{k_y+q_y,k_{y'}}\,\delta\left(\Delta E \mp \hbar\omega_{LO} \mp \hbar\Omega\right), \tag{14}$$

$$\text{where} \quad \Delta E = (N' - N)\,\hbar\omega_c + (E_{n'} - E_n)\,. \tag{15}$$

Following the collision - broadening model, we shall replace the delta function by Lorentzian broadening with the width $\Gamma_{NN'}$ [21]

$$\delta(\Delta E \mp \hbar\omega_{LO} \mp \hbar\Omega) = \frac{1}{\pi}\left[\frac{\hbar\Gamma_{NN'}^{\pm}}{\left[\Delta E \mp \hbar\omega_{LO} \mp \hbar\Omega\right]^2 + (\hbar\Gamma_{NN'}^{\pm})^2}\right], \tag{16}$$

where $(\Gamma_{NN'}^{\pm})^2 = |M_{int}|^2$ as displayed in Eq. (10).

3. RESULTS AND DISCUSSION

In this section, the numerical results for a GaAs/$\mathrm{Ga_{0.7}Al_{0.3}}$As quantum well are presented, for which we take the following parameters: [22, 23] $\varepsilon_\infty = 10.89$, $\varepsilon_0 = 13.18$, $n = 3.2$, $\hbar\omega_{LO} = 36.25$ meV, and the electron density $n_e = 3\times10^{16}$ cm^{-3}. We consider the intrasubband transition of electron ($n = n'$) and between Landau levels $N = 0$ and $N' = 1$. The CPR condition is $\hbar\Omega = p\hbar\omega_c + \hbar\omega_{LO}$.

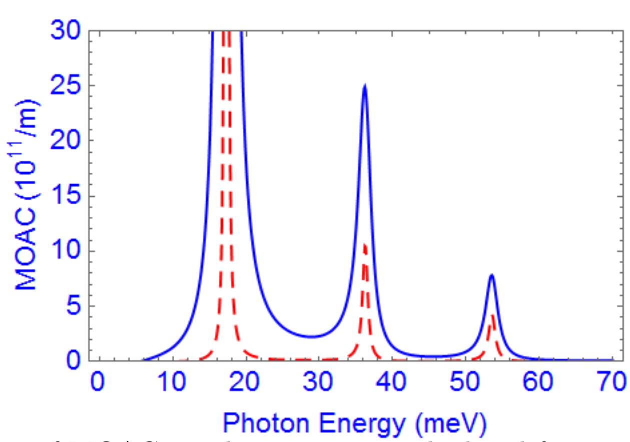

Figure 1: Dependence of MOAC on photon energy calculated for $T = 300$ K, $L_z = 12$ nm, $B = 10$ T. The full curve is for confined phonons described by the HZ model, the dashed curve is for bulk phonons.

Figure 1 shows the variation of the MOAC with the photon energy in the case of confined and bulk phonons. It can be seen from the figure that there exist three resonant peaks in each curve, labelled from "1" to "3". All these peaks are the result of the electron transitions, satisfying the CPR condition, which can be explained as follows:

+ Peak 1 at $\hbar\Omega = 17.29$ meV, satisfies the cyclotron resonance condition $\hbar\Omega = \hbar\omega_c$.

+ Peak 2 at $\hbar\Omega$=36.25 meV, satisfies the condition $\hbar\Omega = \hbar\omega_{LO} = 36.25$ meV.

+ Peak 3 at $\hbar\Omega = 53.54$ meV, satisfies the condition $\hbar\Omega = \hbar\omega_c + \hbar\omega_{LO}$ or $53.53\,\text{meV} = 17.28\,\text{meV} + 36.25\,\text{meV}$. This corresponds to the transition of electron from Landau level $N = 0$ to $N' = 1$ by absorbing one photon with energy $\hbar\Omega$ and simultaneous emitting one LO-phonon of energy $\hbar\omega_{LO}$.

The magnetic field and well width are proved to have an important role in the optical absorption properties of the low-dimensional quantum systems because it modifies the energy separatonΔE. Therefore, it is necessary to study the effect of the magnetic field and well width on the MOAC and the FWHM.

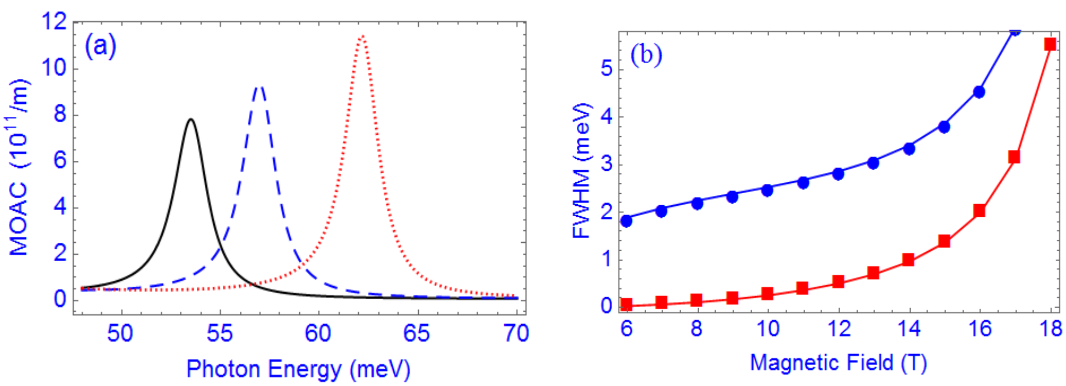

Figure 2: (a) Dependence of MOAC on the photon energy for confined phonons at different values of the magnetic field B: $B = 10$ T (solid line), $B = 12$ T (dashed line) and $B = 15$ T (dotted line) with $L_z = 12$ nm, $T = 300$ K. (b) Dependence of FWHM on magnetic field for bulk phonons (line with squares) and confined phonons (lines with circles).

Figure 2a indicates the dependence of the MOAC on the photon energy at the peak of the CPR for the confined phonon at different values of the magnetic field B. From the figure we see that when the magnetic field B increases, the positions of the resonant peak shift towards the greater photon energy (blue-shift). This can be explained by the fact that when B increases, cyclotron energy $\hbar\omega_c$ increases, causing the energy value of the absorbed photon satisfying the resonance condition $\hbar\Omega = \hbar\omega_c + \hbar\omega_{LO}$ also increases. Moreover, when the magnetic field increases the peak height grows quickly with a nonlinear law. This behavior can be interpreted by the fact that the expression of MOAC (Eq. (14)) proportional to B^2. The results are in remarkable agreement with previous works [24, 25].

Figure 2b shows the dependence of the FWHM on the magnetic field for both cases of bulk phonon and confined phonon. From the figure we see that the FWHM increases when the magnetic field increases. This can be explained that when the magnetic field increases, the cyclotron radius $\ell_B = (\hbar/eB)^{1/2}$ decreases, leading to an increment in electron confinement, so the probability of electron - phonon scattering increases. Therefore, the FWHM increases when the magnetic field increases. This greatly agrees with that obtained in other types of quantum well [26, 27, 28]. In particular, the FWHM for the case of confined phonons is greater than that in the case of bulk phonons. This can be explained that when phonons are confined, the probability of electron - phonon scattering increases. Therefore, when the magnetic field increases, the phonon confinement becomes more important and cannot be ignored. This results is in agreement with that in ref [31].

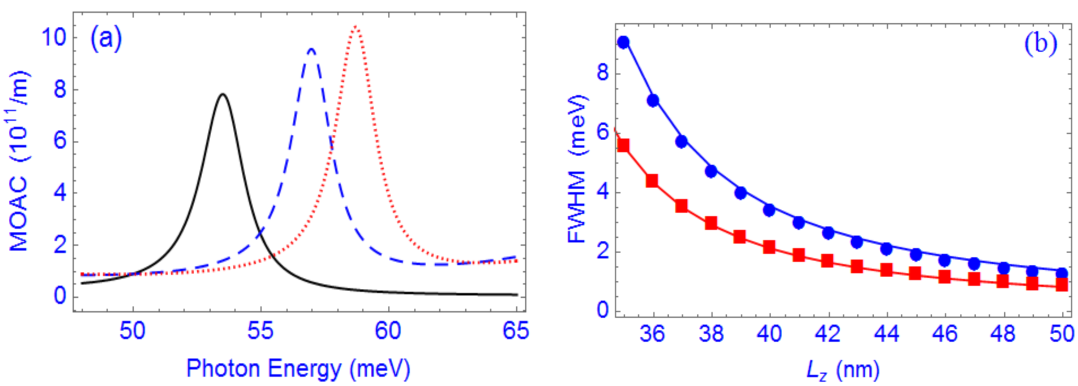

Figure 3: (a) The dependence of the MOAC on the photon energy for confined phonons at different values of the well widthL_z: $L_z = 25$nm (solid line), $L_z = 20$ nm (dashed line) and $L_z = 15$ nm (dotted line) with $B = 10$ T, $T = 300$ K. (b) Dependence of the FWHM on well width for bulk phonons (line with squares) and confined phonons (lines with circles).

Figure 3a shows the dependence of the MOAC on the photon energy at the peak of the CPR for confined phonons at different values of the well width L_z. From the figure we see that when L_z increases, the positions of the resonant peak shift towards the lower photon energy (red-shift). This red-shift behavior observed here is in agreement with that reported in previous works for Pöschl-Teller quantum well [29, 30]. The reason for this is that the energy of the electron is inversely proportional to L_z^2, when L_z increases then the energy separation ΔE decreases, corresponding to the energy value of the absorbed photon decreases accordingly. We also observe that the peak height is lower as L_z becomes larger.

Figure 3b indicates the dependence of the FWHM on the width of the well. From the graph we see that the FWHM increases as L_z decreases for both bulk phonon and confined phonon. This can be explained by the fact that as the well width increases, electron confinement decreases, the probability of electron-phonon scattering decreases, therefore the FWHM decreases as the size of the quantum well increases. This result agrees qualitatively with that in previous works [26, 27, 28], in which the FWHM gets smaller value in the range of wider quantum well. In addition, the FWHM for the case of confined phonons is greater than the case of bulk phonons. This can be explained that when the phonon is confined, the probability of electron - phonon scattering increases, therefore the phonon confinement becomes more important in narrow quantum wells. This results agrees with that showing in ref [31].

4. CONCLUSION

We have presented detailedly the influence of the phonon confinement on the CPR effect in SQW. The phonon confinement is described by the Huang - Zhu model. The results are discussed by considering the role of magnetic field and well width in the change of MOAC and FWHM. Since making a significant change in the energy separation, both these parameters affect sensitively not only the height and position of resonant peaks but also the FWHM. The MOAC peaks shift to higher photon energies when the magnetic field increases, but shift to lower photon energies in the range of wider well. The peak heights are grown by the rise of magnetic field, but reduced with the well width. The FWHM is observed to be a nonlinear function of B and L_z, that means it rises with the magnetic field but reduces with the well-width. In both cases, the FWHM for the confined phonon is always bigger than that for bulk phonons. The results obtained here are in good agreement with previous theoretical and experimental works reported in other quantum well structures. To our understanding, they are new and their validity needs to be verified by future experimentation.

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